Nonlinear evolution at small x with impact parameter dependence

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Abstract. The BK evolution equation was evaluated numerically with full dependence on impact parameter and several distinct behaviors were found. These behaviors are presented for the leading logarithmic kernel in the BK evolution equation with both fixed and running coupling. The saturation scale was found to agree with analytic expectations. The results for the evolution with running coupling were then compared to the HERA data of the F_2 structure function.

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INTRODUCTION

At small values of x_{bj} it is expected that the growth of the number of gluons inside nucleons has to be damped and the system should become saturated. In this saturated regime gluon recombination effects become important and new dynamics become emergent. One result from the study of this saturation region is the Balitsky-Kovchegov (BK) evolution equation (1) [1, 2]. The BK equation is an integro-differential equation which evolves towards smaller x. It was shown that the BK equation can be derived within the dipole model [3], where a virtual photon fluctuates into a $q\bar{q}$ pair before interacting with the nucleon via pomeron exchange. In this model the solution to the BK equation is the scattering amplitude between the color dipole and the nucleon. The evolution towards smaller x can be described as dipole cascade where the incoming (parent) color dipole emits a gluon, effectively splitting into two color dipoles. This splitting is given by the kernel of the BK equation which contains the dynamics of the dipole splitting as well as the running of the coupling α_s. The BK equation has the form

$$\frac{\partial N_{\mathbf{x}_0\mathbf{x}_1}}{\partial Y} = \int \frac{d\mathbf{x}_2^2}{2\pi} K(x_0,x_1,x_2)\left[N_{\mathbf{x}_0\mathbf{x}_2} + N_{\mathbf{x}_1\mathbf{x}_2} - N_{\mathbf{x}_0\mathbf{x}_1} - N_{\mathbf{x}_0\mathbf{x}_2}N_{\mathbf{x}_1\mathbf{x}_2}\right] \tag{1}$$

where N is the scattering amplitude and it is dependent on the coordinates of the dipole involved. The parent dipole is defined by coordinates $\mathbf{x}_0\mathbf{x}_1$ which splits into two daughter dipoles $\mathbf{x}_0\mathbf{x}_2$ and $\mathbf{x}_1\mathbf{x}_2$ with $K(x_0,x_1,x_2)$ defined to be the splitting kernel [1].

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[1] Bold denotes vetor quantities

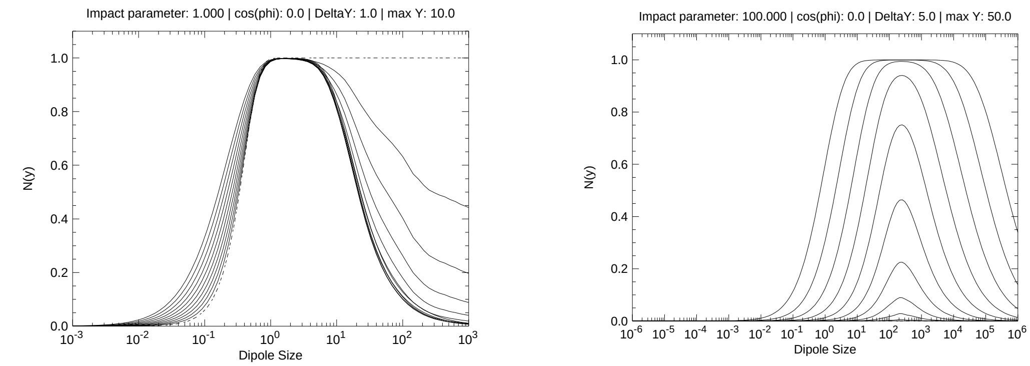

FIGURE 1. Two plots of scattering amplitude versus dipole size for different sizes of impact parameter. The left graph has a impact parameter of 1.00 and each curve indicates one unit of rapidity past the iniital condition to a maximum of ten. The right graph has a impact parameter of 100.00 and each curve is five units of rapidity past the initial condition to a maximum of 50. The initial condition on the right is near zero for this impact parameter. It is clear on the right graph that the evolution equation picks out $r = 2b$ and a peak forms at that point. The appearance of a second evolution front towards large dipole sizes can be seen in addition to the usual evolution front towards small dipole sizes.

BK EQUATION WITH IMPACT PARAMETER DEPENDENCE

There have been several other numerical solutions to the BK equation [4, 5, 6, 7] with most of them neglecting impact parameter. The inclusion of impact parameter has a marked affect [8, 9] on the evolution of the scattering amplitude. The initial analysis was done with the leading logarithmic (LL) kernel (2) and fixed coupling $\frac{\alpha_s N_c}{\pi} = 0.1$.[2]

$$K = \frac{dz}{z} \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \tag{2}$$

Inclusion of impact parameter has the distinct feature of causing the scattering amplitude for large dipole sizes to go to zero. With impact parameter dependence the target is localized and large dipoles miss the target giving us a zero scattering amplitude. This can be seen in Fig:1 where the inital condition which is of a Glauber-Mueller type (3) quickly drops to zero for large dipole sizes.

$$N_{\mathbf{x}_0\mathbf{x}_1} = 1 - e^{-x_{01}^2 e^{b_{01}^2/4}} \tag{3}$$

The graphs in this section have no units as no physical scale was imposed in the problem, this will be done in the next section for comparison with data.

It is also found that when dipole size is equal to twice the impact parameter $\mathbf{x}_{01} = 2\mathbf{b}_{01}$ there is an enhancement in the evolution creating a peak at this position. The reason for this peak was described in terms of the dipole model as well as the conformal eigenfunction representation [10] which is described more in [8].

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[2] $x_{01} = |\mathbf{x}_0 - \mathbf{x}_1|$ and $b_{01} = \frac{1}{2}|\mathbf{x}_0 + \mathbf{x}_1|$

The saturation scale Q_s is the scale at which parton recombination effects become important and is defined as $< N_{\mathbf{x}_0\mathbf{x}_1} >= \kappa$ where $\kappa = 0.5$ is chosen. The saturation scale is parameterized as $Q_s^2 = Q_{s0}^2 e^{\lambda_s Y \bar{\alpha}_s}$ where the exponent λ_s can be extracted from asymtopically high rapidities. It was found that our solution for the evolution with the LL kernel that $\lambda_s = 4.4$ which is in agreement with analytic predictions [11].

Comparison with HERA data

Our numerical solution for the scattering amplitude can be compared to data from H1 and ZEUS [12] by computing the F_2 structure function for the proton.

$$F_2(Q^2,x) = \frac{Q^2}{4\pi^2\alpha_{em}} \int d^2\mathbf{r} \int_0^1 dz \left(|\Psi(r,Q^2,z)_T|^2 + |\Psi(r,Q^2,z)_L|^2\right) \sigma_{bdip}(r,x) \qquad (4)$$

Here $\sigma_{dip}(\mathbf{r},x) = \int d^2\mathbf{b} 2N(\mathbf{r},\mathbf{b},Y)$ and $|\Psi(r,Q^2,x)_{T/L}|^2$ is the probability for the virtual photon to fluctuate into a color dipole [13]. To meaningfully compare the solution of the BK equation with impact parameter to the data running coupling must be taken into account in the kernel. There are two calculations which do this [14, 15] and we chose to implement the one by Balitsky as the Kovchegov-Weigert prescription was found to be more numerically involved to evaluate. We also implemented massive cuts on the kernel in the form of theta functions (5). When the emitted daughter dipoles exceeded a scale $\frac{1}{m}$ the kernel was set to zero, this damped the growth of the amplitude in the non-perturbative large-dipole regime. Several other methods for impliementing these massive cuts were attempted [16] and this method was found to be most consistent with the data.

$$K = \frac{N_c \alpha_s(x_{01}^2)}{2\pi^2} \left[\frac{1}{x_{02}^2} \left(\frac{\alpha_s(x_{02}^2)}{\alpha_s(x_{12}^2)} - 1 \right) + \frac{1}{x_{12}^2} \left(\frac{\alpha_s(x_{12}^2)}{\alpha_s(x_{02}^2)} - 1 \right) + \frac{x_{01}^2}{x_{12}^2 x_{02}^2} \right] \Theta(\frac{1}{m^2} - x_{02}^2) \Theta(\frac{1}{m^2} - x_{12}^2) \qquad (5)$$

The coupling constant was chosen to be regulated by adding in a mass parameter μ into the coupling as $\alpha_s(x^2) = \frac{1}{bln\left[\Lambda^{-2}\left(\frac{1}{x^2}+\mu^2\right)\right]}$. The regularization of the coupling is more important with impact parameter dependence due to the second evolution front at large dipole size which is highly dependent on the coupling in this regime.

The initial condition was taken from [17] which was fit to the F_2 data and $Y = 0$ was set to $x_{bj} = 10^{-2}$. The Balitsky kernel slows the evolution down a dramatic amount which allows it to qualitativly fit the data. The calculation is below the data, as can be seen in Fig:2, because the mass cuts remove a non-trivial contribution from very large dipole sizes. This contribution must be added in seperately as either a fit term, a vector meson dominance (VMD) term or a combination of the two. These additional terms were added in [16].

The evolution with Balitsky kernel was found to be very sensitive to the regularization details. This could be attributed to the highly nonlinear form of this kernel, but more analysis should be performed in order to understand this sensitivity

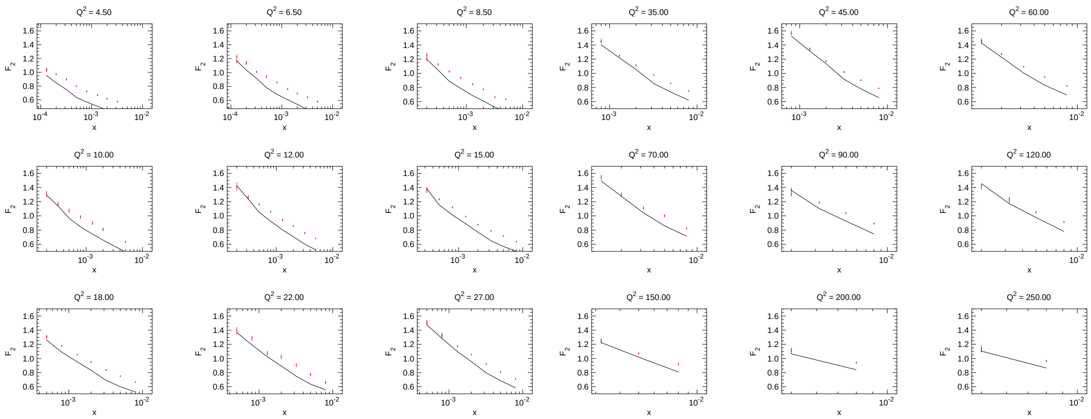

FIGURE 2. Plots of F_2 comparing the solution of the BK equation with the Balitsky kernel with parameter values of $\mu = 0.52 GeV$ $m = 350 MeV$ to combined HERA data [12].

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